THE REBINDER NUMBER AS A CHARACTERISTIC OF THE THERMODYNAMIC AND TRANSPORT PROPERTIES OF DISPERSED MATERIALS

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An equation connecting the Rebinder number with the thermodynamic and transport characteristics of dispersed materials and the parameters of external heat and mass transfer is obtained.

The Rebinder number Rb and the dimensionless temperature coefficient of drying B, introduced by Lykov in the form of the relationships [1]

$$\mathrm{Rb} = \frac{c_q d\bar{t}}{r d\bar{u}}, \quad B = \frac{d\bar{t}\,(\bar{u} - u_0)}{d\bar{u}\,(\bar{t} - t_0)} = \mathrm{Rb\,Ko}, \tag{1}$$

are the main characteristics of the material in the equation representing the drying kinetics of dispersed materials:

$$\mathrm{Ki}_q = \mathrm{Ki}_m(\tau)\,\mathrm{Lu\,Ko}\,(1 + \mathrm{Rb}) = \mathrm{Ki}_m(\tau)\,\mathrm{Lu}\,(\mathrm{Ko} + \mathrm{B}). \tag{2}$$

The parameters Rb and B are characteristics of the integral equation of drying kinetics and, hence, they take into account the properties of the material and its interaction with the surrounding medium. It is of interest to establish a relationship between the integral parameters Rb and B and the local characteristics – the criteria of external and internal heat and mass transfer. The present paper is devoted to this important current problem.

Convective drying of a dispersed material is represented by a system of nonlinear equations [1]:

$$\frac{\partial t}{\partial \tau} = \nabla (a_q \nabla t) + \frac{\varepsilon r}{c_q} \frac{\partial u}{\partial \tau}, \tag{3}$$

$$\frac{\partial u}{\partial \tau} = \nabla (a_m \nabla u + a_m \delta \nabla t) \tag{4}$$

with boundary conditions of the third kind:

$$\lambda_q (\nabla t)_s = \alpha_q (t_0 - t_s) + (1 - \varepsilon) r \alpha_m' (u_0 - u_s), \tag{5}$$

$$a_m \gamma (\nabla u)_s + a_m \gamma \delta (\nabla t)_s = \alpha_m (u_0 - u_s). \tag{6}$$

In principle the solution of system (3)-(4), taken in conjunction with (5)-(6), can be used to find $\bar{t}$ and $\bar{u}$, and then Rb and B can be found as functions of the material characteristics (a_q, a_m, δ, c_q, r) and drying conditions (α_q, α_m, t_0, u_0) from Eqs. (1). Mathematical difficulties, however, prevent a complete analytical solution of this problem. The best attempt has been the obtention of a numerical solution on a computer, as in [2], for instance, where all the transport coefficients were assumed constant, which is a rather coarse approximation [1, 3]. Moreover, numerical solutions are not suitable for the investigation of Rb and B in relation to particular factors. Hence, it is better to choose another approach.

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By definition:

$$\bar{t}=\frac{1}{V}\int_V t\,dV,\quad \bar{u}=\frac{1}{V}\int_V u\,dV. \tag{7}$$

Applying (7) to (3) and (4) and using the Gauss theorem, we obtain:

$$\frac{d\bar{t}}{d\tau}=\frac{1}{V}\int_F a_q(\nabla t)_s dF+\frac{\varepsilon r}{c_q}\frac{d\bar{u}}{d\tau}, \tag{8}$$

$$\frac{d\bar{u}}{d\tau}=\frac{1}{V}\int_F [a_m(\nabla u)_s+a_m\delta(\nabla t)_s]\,dF, \tag{9}$$

from which after integration:

$$\frac{d\bar{t}}{d\tau}=\frac{F}{V}a_q(\nabla t)_s+\frac{\varepsilon r}{c_q}\frac{d\bar{u}}{d\tau}, \tag{10}$$

$$\frac{d\bar{u}}{d\tau}=\frac{F}{V}a_m(\nabla u)_s+\frac{F}{V}a_m\delta(\nabla t)_s. \tag{11}$$

where t_s and u_s are quantities averaged over the surface.

We note that in (5) the phase change number ε, like all the other characteristics of the material, relates to the surface of the material. In this case ε for all materials is unity, since in convective drying mass can be removed from the surface of the material only in the form of vapor. A change in the mean (integral) mass content $\bar{u}$ of the material in convective drying can also be effected only by evaporation. The transfer of mass within the material in liquid form, which corresponds to $\varepsilon < 1$, leads only to a redistribution of the mass content within the material and $\bar{u}$ remains constant. Hence, in (8) and (10) [but not in (3)] ε is also unity.

For simplicity we will consider the one-dimensional symmetric problem – the drying of an infinite plate of thickness $2l$ with the coordinate origin at the center of the plate. The ratio V/F in (10) and (11) is then equal to the plate thickness l. We assume now that the distributions of temperature and mass content in the material are given by:

$$\begin{aligned} t&=t_c-\varphi_q(x)(t_c-t_s),\\ u&=u_c-\varphi_m(x)(u_c-u_s), \end{aligned} \tag{12}$$

where φ_q and φ_m are dimensionless functions satisfying the conditions:

$$(\varphi_q)_c=(\varphi_m)_c=0,\quad (\varphi_q)_s=(\varphi_m)_s=1. \tag{13}$$

We then easily obtain:

$$\begin{aligned} (\Delta t)_s&=\frac{(\nabla\varphi_q)_s(\bar{t}-t_s)}{\frac{1}{V}\int_V \varphi_q dV-1}=\frac{k_q}{l}(\bar{t}-t_s),\\ (\nabla u)_s&=\frac{(\nabla\varphi_m)_s(\bar{u}-u_s)}{\frac{1}{V}\int_V \varphi_m dV-1}=\frac{k_m}{l}(\bar{u}-u_s), \end{aligned} \tag{14}$$

where k_q and k_m are dimensionless coefficients which depend on the distribution of t and u in the material.

Substituting (14) in (10) and (11) and using the boundary conditions (5) and (6) we eliminate t_s and u_s from the equations and obtain:

$$\frac{d\bar{t}}{d\tau}=\frac{r\,d\bar{u}}{c_q\,d\tau}+\frac{a_qk_q\alpha_q(t_0-\bar{t})}{l(\alpha_qk_q+\alpha_ql)}, \tag{15}$$

$$\frac{d\bar{u}}{d\tau}=\frac{a_mk_m\alpha_m(\lambda_qk_q+\alpha_ql)(u_0-\bar{u})+a_m\delta k_ql\alpha_q\alpha_m(t_0-\bar{t})}{l(\lambda_qk_q+\alpha_ql)(a_m\gamma k_m+\alpha_ml)}. \tag{16}$$

After substituting (15) and (16) in (1) we finally obtain:

$$\mathrm{Rb} = 1 + \frac{c_q a_q k_q \alpha_q \left(a_m \gamma k_m + \alpha_m l\right)}{r a_m k_m \alpha_m \left[\left(\lambda_q k_q + \alpha_q l\right) \dfrac{u_0 - \bar{u}}{t_0 - \bar{t}} + \dfrac{k_q}{k_m}\, \delta l \alpha_q\right]}, \tag{17}$$

$$B = \frac{r\left(\bar{u} - u_0\right)}{c_q\left(\bar{t} - t_0\right)} + \frac{a_q k_q \alpha_q \left(a_m \gamma k_m + \alpha_m l\right)}{a_m k_m \alpha_m \left[\left(\lambda_q k_q + \alpha_q l\right) + \delta \alpha_q l\, \dfrac{k_q \left(u_0 - \bar{u}\right)}{k_m \left(t_0 - \bar{t}\right)}\right]}. \tag{18}$$

Equations (17) and (18) express Rb and B as functions of the material characteristics and the parameters of external heat and mass transfer. It is important here that no assumptions regarding the constancy of the transport coefficients were made in the deduction of (17) and (18). Hence, (17) and (18) are valid also for significantly nonlinear heat- and mass-transfer processes, a typical example of which is drying.

The only assumption used in the deduction was the introduction of coefficients k_q and k_m characterizing the distribution of t and u in the material. For a parabolic distribution, which follows from the solution of Eqs. (3)-(4) with constant coefficients [4], and has also been confirmed experimentally [5-7], we easily obtain

$$\varphi_q = \varphi_m = \frac{x^2}{l^2}, \tag{19}$$

from which

$$k_q = k_m = 3. \tag{20}$$

For other distributions k_q and k_m are of the same order of magnitude and are equal if $\varphi_q = \varphi_m$. For instance, for a linear distribution of t and u $k_q = k_m = l$, for a cubic distribution $k_q = k_m = 4$, and so on. In the general case k_q and k_m are dimensionless characteristics of the material, since the distribution of t and u in the material is determined by its thermodynamic and transport properties. In particular, the distributions of t and u are affected by the phase change number ε within the material [4], which is not contained in explicit form in the solutions of (17) and (18) for the reasons mentioned above.

Formulas (17) and (18) can be put in another form. Using the Posnov number Pn, given by the relation

$$\mathrm{Pn} = \frac{\delta\left(t_s - t_c\right)}{\left(u_s - u_c\right)}, \tag{21}$$

we obtain

$$\frac{(\nabla t)_s}{(\nabla u)_s} = \frac{(\nabla \varphi_q)_s \,\mathrm{Pn}}{(\nabla \varphi_m)_s \delta} = \frac{k_{qm}}{\delta}\,\mathrm{Pn}, \tag{22}$$

where $k_{qm} = 1$ when $\varphi_q = \varphi_m$. Then (6) and (11) take the form:

$$a_m \gamma \left(1 + k_{qm}\,\mathrm{Pn}\right)(\nabla u)_s = \alpha_m \left(u_0 - u_{\text{п}}\right), \tag{23}$$

$$\frac{d\bar{u}}{d\tau} = \frac{a_m}{l}\left(1 + k_{qm}\,\mathrm{Pn}\right)(\nabla u)_s. \tag{24}$$

Performing all the algebra similar to that described above, but with (6) and (11) replaced by (23) and (24), we obtain expressions for the rate of change of $\bar{t}$ and $\bar{u}$ for Rb and B:

$$\frac{d\bar{t}}{d\tau} = \frac{r\,d\bar{u}}{c_q\,d\tau} + \frac{a_q k_q \alpha_q \left(t_0 - \bar{t}\right)}{l\left(\lambda_q k_q + \alpha_q l\right)}, \tag{25}$$

$$\frac{d\bar{u}}{d\tau} = \frac{a_m k_m \alpha_m \left(1 + k_{qm}\,\mathrm{Pn}\right)\left(u_0 - \bar{u}\right)}{l\left[a_m \gamma k_m \left(1 + k_{qm}\,\mathrm{Pn}\right) + \alpha_m l\right]}, \tag{26}$$

$$\mathrm{Rb} = 1 + \frac{c_q a_q \alpha_q k_q \left[a_m \gamma k_m \left(1 + k_{qm}\,\mathrm{Pn}\right) + \alpha_m l\right]\left(t_0 - \bar{t}\right)}{r a_m \alpha_m k_m \left(1 + k_{qm}\,\mathrm{Pn}\right)\left(\alpha_q l + \lambda_q k_q\right)\left(u_0 - \bar{u}\right)}, \tag{27}$$

$$B = \frac{r\left(\bar{u} - u_0\right)}{c_q\left(\bar{t} - t_0\right)} + \frac{a_q k_q \alpha_q \left[a_m \gamma k_m \left(1 + k_{qm}\,\mathrm{Pn}\right) + \alpha_m l\right]}{a_m k_m \alpha_m \left(1 + k_{qm}\,\mathrm{Pn}\right)\left(\lambda_q k_q + \alpha_q l\right)}. \tag{28}$$

In dimensionless-number form (27) and (28) become:

$$\mathrm{Rb} = 1 + \frac{k_q \mathrm{Bi}_q [k_m (1 + k_{qm} \mathrm{Pn}) + \mathrm{Bi}_m]}{k_m \mathrm{Bi}_m \mathrm{Lu}\, \mathrm{Ko} (1 + k_{qm} \mathrm{Pn}) (k_q + \mathrm{Bi}_q)}, \tag{29}$$

$$B = \mathrm{Ko} + \frac{k_q \mathrm{Bi}_q [k_m (1 + k_{qm} \mathrm{Pn}) + \mathrm{Bi}_m]}{k_m \mathrm{Bi}_m \mathrm{Lu} (1 + k_{qm} \mathrm{Pn}) (k_q + \mathrm{Bi}_q)}, \tag{30}$$

where the similarity criteria are selected from the relations

$$\mathrm{Lu} = \frac{a_m}{a_q}, \quad \mathrm{Bi}_q = \frac{\alpha_q l}{\lambda_q}, \quad \mathrm{Bi}_m = \frac{\alpha_m l}{a_m \gamma}, \quad \mathrm{Ko} = \frac{r (\bar{u} - u_0)}{c_q (\bar{t} - t_0)}. \tag{31}$$

Finally, for a parabolic distribution of t and u in the material, which is assumed in most investigations, we obtain:

$$\mathrm{Rb} = 1 + \frac{\mathrm{Bi}_q (3 + 3\,\mathrm{Pn} + \mathrm{Bi}_m)}{\mathrm{Bi}_m \mathrm{Lu}\, \mathrm{Ko} (1 + \mathrm{Pn}) (3 + \mathrm{Bi}_q)}, \tag{32}$$

$$B = \mathrm{Ko} + \frac{\mathrm{Bi}_q (3 + 3\,\mathrm{Pn} + \mathrm{Bi}_m)}{\mathrm{Bi}_m \mathrm{Lu} (1 + \mathrm{Pn}) (3 + \mathrm{Bi}_q)}. \tag{33}$$

Equations (17), (18) and (29), (30) express in explicit form Rb and B as functions of the characteristics of the dispersed material (a_q, a_m, c_q, r, δ, Lu, Pn) and the parameters of the external heat and mass transfer (α_m, α_q, t_0, u_0, Bi_q, Bi_m). In particular, it is clear from (29) and (30) that the effect of the drying regime parameters on Rb and B is determined mainly by the relation between the internal heat and mass transfer (λ_q and $a_m\gamma$) and the external heat and mass transfer (α_m and α_q), i.e., by the value of the Biot number.

An analysis of Eqs. (17), (18) and (29), (30) can provide an answer to the two main questions associated with the practical application of Rb and B. Firstly, the obtained solutions indicate the experimental conditions in which the measured Rb and B are single-valued characteristics of the dispersed material and are independent of the prescribed drying conditions. Secondly, a relationship can be established between the Rb and B which are characteristics of the material, and the Rb and B which are contained in the equation of convective drying in particular prescribed conditions. To solve these questions we consider the behavior of Eqs. (17), (18) and (29), (30) in different regimes.

We consider the limiting case of drying of "thick" samples in a comparatively rapid drying regime, where $\mathrm{Bi}_q \to \infty$, $\mathrm{Bi}_m \to \infty$. In this case we obtain from (17), (18) and (29), (30).

$$\mathrm{Rb}^0 = 1 + \frac{c_q a_q k_q (t_0 - \bar{t})}{r a_m [k_m (u_0 - \bar{u}) + k_q \delta (t_0 - \bar{t})]} \tag{34}$$

or

$$\mathrm{Rb}^0 = 1 + \frac{k_q}{k_m \mathrm{Lu}\, \mathrm{Ko} (1 + k_{qm} \mathrm{Pn})} \tag{35}$$

and

$$B^0 = \frac{r (u_0 - \bar{u})}{c_q (t_0 - \bar{t})} + \frac{a_q k_q (t_0 - \bar{t})}{a_m [k_m (u_0 - \bar{u}) + k_q \delta (t_0 - \bar{t})]} \tag{36}$$

or

$$B^0 = \mathrm{Ko} + \frac{k_q}{k_m \mathrm{Lu} (1 + k_{qm} \mathrm{Pn})}. \tag{37}$$

At high values of the Biot number $t_s \to t_0$ and $u_s \to u_0$. In view of this, according to (14), (21), (22), and (31), we can write:

$$\frac{k_m \mathrm{Ko}}{k_q} = \frac{\mathrm{Fe}'}{k_{qm} \mathrm{Pn}}, \tag{38}$$

where

$$\mathrm{Fe}' = \mathrm{Ko\,Pn} = \frac{\delta r}{c_q} \tag{39}$$

depends only on the properties of the material. In exactly the same way Pn in the considered regime depends only on the internal heat and mass transfer in the material. From (34) we then obtain

$$\mathrm{Rb}^0 = 1 + \frac{k_{qm}\,\mathrm{Pn}}{\mathrm{Lu\,Fe}'\,(1 + k_{qm}\,\mathrm{Pn})}. \tag{40}$$

It is apparent from (40) that Rb^0 depends only on quantities characterizing the internal heat and mass transfer in the material. In other words, Rb, measured in this drying regime, is a characteristic of the thermodynamic (c_q, r) and transport (a_m, a_q, δ, k_q, k_m, k_{qm}) properties of the material and is completely independent of the conditions of external heat and mass transfer. This criterion can conveniently be designated Rb^0.

It is clear from what has been said that an investigation of the heat- and mass-transfer properties of dispersed materials and the classification of these materials according to their behavior in the drying process will have to be based on Rb^0, and not Rb, measured in particular conditions, since it is Rb^0 which characterizes the properties of the material, which are independent of the arbitrarily prescribed drying regime parameters.

The use of Rb^0 as a characteristic of the transport properties of dispersed materials is particularly advantageous in cases where the transport coefficients depend strongly on the temperature and mass content. In fact, the overwhelming majority of experimental methods of determining a_m and δ are based on solutions of Eqs. (3) and (4) with piecewise-constant coefficients, which introduces a large error into the theoretical equations [1, 3] and makes such methods fundamentally nonrigorous. At the same time, the experimental measurement of Rb^0 from Eq. (1) is effected without any assumptions and is comparatively easy. The accuracy of this measurement is limited only by the accuracy of the measuring instruments. Thus, Rb^0 can be used as a characteristic of the transport properties of dispersed materials on a par with the usually employed characteristics (a_q, a_m, δ). Its use is often much more convenient, since Rb^0 can be experimentally measured more easily and more accurately than the other characteristics.

Equally important is the establishment of a relationship between Rb characteristic of a particular drying regime and Rb^0, since the basic drying Eq. (2) contains Rb, and not Rb^0. In the general case this relationship is given by Eqs. (27), (29) and (34), (40) and is complex. The introduction of some assumptions gives an approximate relationship in a simpler form.

For arbitrary Bi_q and Bi_m Pn depends not only on the properties of the material, but also on the drying regime. If, as a first approximation, we assume that Pn depends weakly on the drying regime, then from (29) and (34) we obtain

$$\mathrm{Rb} = 1 + \frac{c_q a_q k_q\,(t_0 - \bar{t})\left[1 + \dfrac{k_m\,(1 + k_{qm}\,\mathrm{Pn})}{\mathrm{Bi}_m}\right]}{r a_m k_m\,(1 + k_{qm}\,\mathrm{Pn})(u_0 - \bar{u})\left(1 + \dfrac{k_q}{\mathrm{Bi}_q}\right)}, \tag{41}$$

from which

$$\frac{\mathrm{Rb} - 1}{\mathrm{Rb}^0 - 1} = \frac{1 + \dfrac{k_m\,(1 + k_{qm}\,\mathrm{Pn})}{\mathrm{Bi}_m}}{1 + \dfrac{k_q}{\mathrm{Bi}_q}}. \tag{42}$$

In principle the measurement of Bi_q and Bi_m is fairly complicated. In convective drying, however, Bi_q and Bi_m usually depend equally on the blowing rate. Hence, we can expect that Rb and Rb^0 will be approximately equal even in cases where the numerator and denominator in (42) are far from unity. This is confirmed by the results of [8], where it was shown experimentally that Rb is not greatly affected by the drying regime parameters for different values of Bi_q, even values greater than unity. The approximate numerical calculation of Rb made in [2] also shows that Rb is practically independent of Bi_q when Bi_q is large.

We consider finally the case where Bi_q and Bi_m are less than unity, which corresponds to gentle drying of a thin layer of material. It follows directly from (27) that

$$\mathrm{Rb} = 1 + \frac{\alpha_q\left(1 + \frac{\mathrm{Bi}_m}{k_m}\right)(t_0 - \bar{t})}{r\alpha_m\left(1 + \frac{\mathrm{Bi}_q}{k_q}\right)(u_0 - \bar{u})}. \quad (43)$$

If Bi_q is small, then, expanding $(1 + Bi_q/k_q)^{-1}$ in a series and taking only the first term we can write:

$$\mathrm{Rb} = 1 + \frac{\alpha_q(t_0 - \bar{t})}{r\alpha_m(u_0 - \bar{u})}\left(1 + \frac{\mathrm{Bi}_m}{k_m} - \frac{\mathrm{Bi}_q}{k_q}\right). \quad (44)$$

In the limiting case where $Bi_q = Bi_m = 0$ all the characteristics of the material disappear from (43) and we obtain the trivial equality

$$\mathrm{Rb} = 1 + \frac{\alpha_q(t_0 - \bar{t})}{r\alpha_m(u_0 - \bar{u})} = 0, \quad (45)$$

since for a thin layer

$$\alpha_q(t_0 - \bar{t}) + r\alpha_m(u_0 - \bar{u}) = 0. \quad (46)$$

The considered limiting cases of small and large values of the Biot number do not, of course, represent the whole region of application of the obtained equations. These equations can probably be simplified considerably for practical calculations, but the solution of this question will require experimental data, which are still very scarce at present.

NOTATION

u	is the mass content, kg/kg;
t	is the temperature, °K;
τ	is the time, sec;
a_q, a_m	are the heat- and mass-diffusion coefficients, m^2/sec;
ε	is the phase change number;
r	is the specific heat of evaporation, J/kg;
c_q	is the specific heat, J/kg·deg K;
δ	is the thermogradient coefficient, 1/deg K;
λ_q	is the thermal conductivity, W/m·deg K;
γ	is the density, kg/m^3;
α_q	is the external heat-transfer coefficient, W/m^2·deg K;
α_m	is the external mass-transfer coefficient, kg/m^2·sec;
V, F, l	are the volume, surface, and thickness, respectively, of sample.

Subscripts

c	denotes the center of sample;
s	denotes the surface of sample;
0	denotes the equilibrium state.

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